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**NATIONAL TECHNICAL UNIVERSITY OF UKRAINE**

**“IGOR SIKORSKY KYIV POLYTECHNIC INSTITUTE”**

# PHYSICS:

# PHYSICAL PRACTICUM IN ELECTROMAGNETISM:

## Laboratory works

Approved by the by the Methodical Board of
the Igor Sikorsky Kyiv Polytechnic Institute
as a study aid for the foreign students
for the specialties 131 Applied mechanics; 133 Manufacturing engineering
134 Aviation, rocket and space machinery; 173 Avionics
of the Institute of Mechanical Engineering

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Electronic Publication

# PHYSICS:

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# ANNOTATION
## For the educational publication
## “Physics: Physical practicum in Electromagnetism: Laboratory works”

Methodical recommendations for laboratory works in Physics for students who study section "Electromagnetism" in physics and are under the Bachelor’s degree study program the for the specialties 131 Applied mechanics; 133 Manufacturing engineering; 134 Aviation, rocket and space machinery; 173 Avionics of the Institute of Mechanical Engineering. It also could be used for other students’ specialties at the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”.

Methodical recommendations for laboratory works in Physics are designed for the foreign students and written in English, they are understandable and at the same time they correspond to the "Physics" course curriculum for the Institute of Mechanical Engineering by the level of material presentation. Methodical recommendations are a practical guide for performing laboratory works in the laboratories of the Faculty of Physics and Mathematics. They provide students with an opportunity to get acquainted with fundamental laws of physics and to verify directly the implementation of these laws in experiments, to form a sufficient level of competence for carrying out physical experiments, processing data and estimating results.

There are laboratory works from the section "Electromagnetism" in the present publication, namely, such topics as "Determination of the work function of metal", "Study of the hysteresis of ferromagnetic materials", "Investigation of damped oscillations in the oscillating circuit", "Investigation of driven oscillations in the oscillating circuit".

The text of the protocol of each laboratory work is accompanied by necessary explanations, illustrations, tables, description of the experimental setup, the procedure order and processing of the experimental results, control questions.

# Laboratory work № PPE-06

## Determination of the work function of metal

**Objective:** to plot and study the current–voltage characteristic of the vacuum tube diode; to investigate the dependence of the saturation current density of the thermionic emission on the temperature of the cathode and to determine the work function of wolfram.

**Equipment** (Figure 6.1): ДЖ is the power supply; ФПЕ-06 is the cassette with the electric circuit; PA is ammeter, PV is voltmeter. The electric diagram of the experiment is shown in Fig. 6.2.

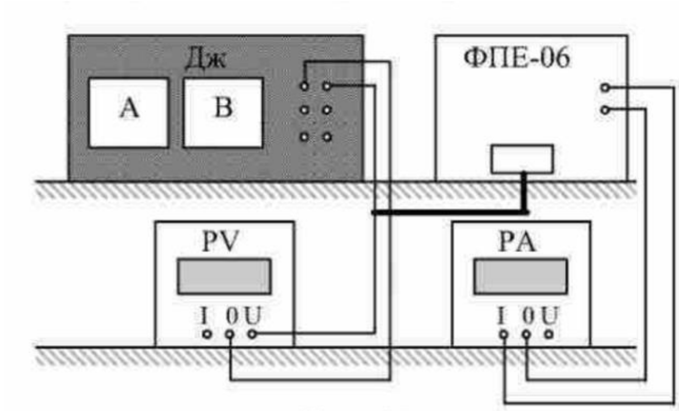



Figure 6.1

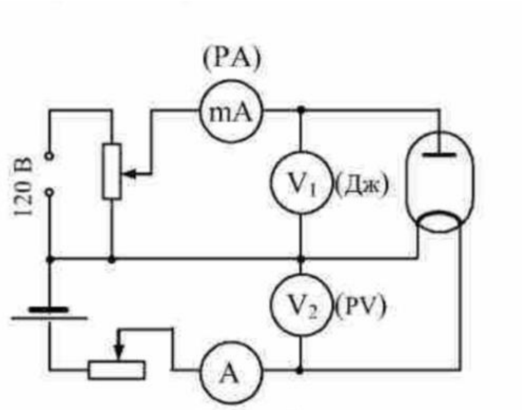



Figure 6.2

In the present work, a directly heated vacuum tube diode with a wolfram cathode is used. The cathode is heated by a constant current. The electric power supplied for the cathode heating is determined using the ammeter and voltmeter.

### Theoretical information

Properties of metals are to a large extent determined by the state of conduction electrons, that is, by electrons capable to move freely inside the metal under the influence of weak electric field. From band theory, there are one or two electrons per atom in a metal that are free to move from atom to atom. This is referred to as a “sea of electrons” or “electron gas”. Their velocities follow a statistical distribution, rather than being uniform, and occasionally an electron will have enough velocity to exit the metal without being pulled back in. The minimum amount of energy needed for an electron to leave a surface is called the **work function**. The work function is characteristic of the material and for most metals is on the order of several electronvolts.

The distribution of the energy states of electron for the bounded metal is shown in the energy diagram (Fig. 6.3). Energy of a free electron with zero kinetic energy outside the metal is taken as zero level.

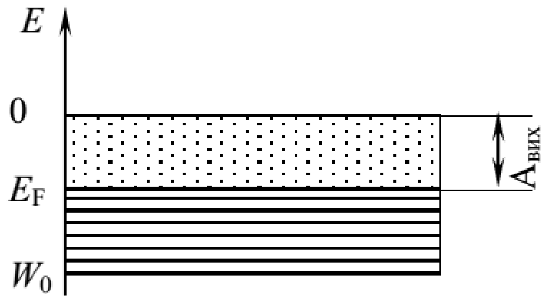



Figure 6.3

The energy levels of electrons are indicated by thin solid horizontal lines that fill the energy interval from the bottom of the potential well $W_0$ to the energy $E_F$ which is called the Fermi energy. The **Fermi energy** is highest occupied single-particle states with maximum kinetic energy at absolute zero temperature $T = 0$.

In order to move beyond the surface of the metal, electrons in the potential well need additional energy. The **work function** of the metal is the minimum kinetic energy needed to remove an electron from a solid to a point in the vacuum immediately outside the solid surface at $T = 0$.

$$A_{вих} = W_0 - E_F$$

At temperature $T$ electrons are in thermal motion, therefore some thermal energy is added to their Fermi energy.

The work function is not a characteristic of a bulk material, but rather a property of the surface of the material (depending on a crystal face and impurities). The Fermi energy practically does not change when a metal is heated below the melting point. However, a certain number (a small percentage) of fast electrons appears that is capable of overcoming the work function of the metal and thus can leave the surface. Consider the nature of forces that prevent electrons from leaving the metal surface, that is, forces creating a certain potential energy barrier for electrons.

After emission of electrons from the surface, a positive charge that is equal in magnitude to the total negative charge emitted is initially left behind in the emitting region. The emitted electrons move away from the surface until the Coulomb interaction with the generated positive charges causes them to come back to the metal.

The process of emission of electrons can be compared with evaporation of molecules from a surface of a liquid: continuously some electrons "evaporate" from the surface of the metal, while others come back. Therefore, an electron cloud is formed near the surface of the metal. The metal is surrounded by the electron cloud, which, along with the surface layer of positive ions, forms a double electric layer similar to a parallel plate capacitor. The interaction of electrons with ions inside of the conductor and within the double electric layer prevents emission of electrons from the metal.

When an electron is emitted from the metal, an induced positive charge immediately appears on the surface of the metal due to the phenomenon of **electrostatic induction** (electrostatic induction is a redistribution of electrical charge in an object, caused by the influence of nearby charges). This induced charge exerts Coulomb force on the electrons counteracting the process of emission. This force is called the force of electric image, since action of the induced charge distributed over the surface of the conductor is equivalent to the action of positive charge of magnitude equal to the charge of emitted electrons and appearing as a mirror image of that electrons (see Figure 6.4).

All of the above physical processes determine the work function $A_{вих}$ of the metal.

At room temperature, almost all free electrons are "closed" within the volume of conductor, and only a small number of electrons with energy sufficient to overcome the potential barrier can be emitted. However, if electrons are provided with additional energy, some of them will be able to leave the metal. This phenomenon is called the thermionic emission.

**Thermionic emission** is the thermally induced flow of electrons from a surface of a metal when the thermal energy given to the electrons is sufficient to overcome the work function of the material. The classical example of thermionic emission is that of electrons from a hot cathode into a vacuum (also known as thermal electron emission or the Edison effect) in a vacuum tube. Vacuum emission from metals tends to become significant only for temperatures over 1000 K.

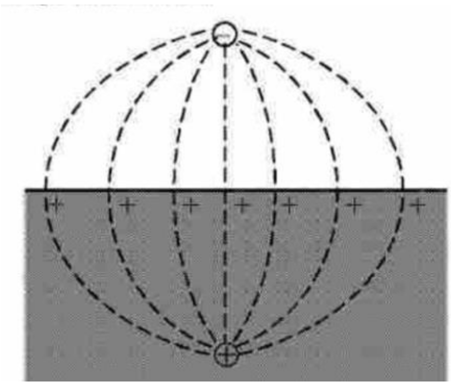


Figure 6.4

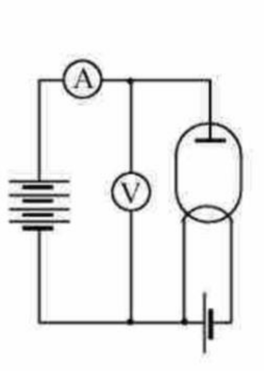



Figure 6.5

To study the thermionic emission, a vacuum tube diode is used. It has two electrodes: a cathode, which is heated by a current, and an anode, which is a cold electrode collecting thermoelectrons.

Figure 6.5 shows the electrical circuitry for such a diode connected with the battery. The thermionic current through the diode depends on the electric potential of the anode relative to the cathode. A graph depicting the relationship between the electric current through the diode and the corresponding voltage across it is called a **current–voltage characteristic** or **I–V curve** (current–voltage curve). Figure 6.6 shows the I–V curve of the vacuum tube diode at different temperatures of the cathode.

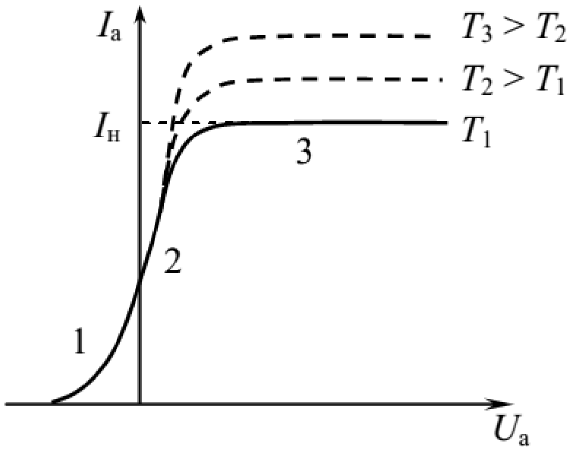



Figure 6.6

When electric potential of the anode is zero, the current is small, it is determined only by the fastest electrons that can reach the anode. As the positive potential of the anode increases, the current increases until it reaches **saturation** when the current does not depend on the voltage any more.

When the temperature of the cathode increases, the value of the current, at which saturation is reached, increases as well. At the same time, the corresponding anode voltage increases too.

Thus, the I-V curve of the diode turns out to be nonlinear, that is, the Ohm's law is not fulfilled. This is explained by the fact that during the thermionic emission the electron cloud with large concentration of electrons is formed near the surface of the cathode. They form a spatial negative charge, and electrons emitted at low speeds can not overcome it. With the increase of the anode voltage, the concentration of electrons in the cloud of spatial charge decreases. Therefore, the braking action of the spatial charge decreases too, so the anode current increases with the increase of the anode voltage in over-linear way.

Theoretically, the dependence of the anode current on the anode voltage in the region 1-2 of the curve on the Figure 6.6 was obtained by Langmuir; it is also called "the law of 3/2":

$$I_a = AU_a^{3/2} \tag{6.1}$$

The increase in the anode voltage leads to the fact that the greater amount of electrons emitted from the cathode is "pulled" to the anode. At a certain value of $U_a$, all the electrons emitted from the cathode per unit time reach the anode. The further growth of the anode voltage can not increase the anode current, hence the saturation is achieved. The maximum thermoelectric current possible at a given temperature of the cathode is called the **saturation current**.

As the temperature increases, the speed of chaotic motion of electron in the metal increases. Moreover, the number of electrons capable of leaving the metal is rapidly increasing. The saturation current density $j_{sat}$ is determined by the **Richardson's law**:

$$j_{sat} = BT^2 e^{-A_{вых}/kT}, \tag{6.2}$$

where $B$ is the emission constant which is different for different metals (for the wolfram $B = 60.2 \cdot 10^4$ A/(m$^2 \cdot$K$^2$)), $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant. The saturation current density characterizes the emission potential of the cathode, which depends on its nature and temperature. Thermionic currents can be increased by decreasing the work function of the metal by applying various oxide coatings to the wire.

## Methodology of the experiment

By measuring dependence of the saturation current on the temperature, we can determine the work function of a given metal.

In such a case, the method of Richardson's lines is used to determine the work function. If we logarithm the expression (6.2), we obtain

$$\ln\frac{j_{sat}}{T^2} = \ln B - \frac{A_{вих}}{kT}, \tag{6.3}$$

or, in decimal logarithms:

$$\lg\frac{j_{sat}}{T^2} = \lg B - \frac{A_{вих}}{k}\frac{1}{T}\lg e \tag{6.4}$$

Since lg$e$ = 0.43, we have:

$$\lg\frac{j_{sat}}{T^2} = \lg B - 0.43\frac{A_{вих}}{k}\frac{1}{T} \tag{6.5}$$

The graph of dependence of $\lg\frac{j_{sat}}{T^2}$ on $1/T$ is a straight line with a slope (angular coefficient) equal to $0.43\frac{A_{вих}}{k}$. This allows us to calculate the work function of the metal with help of the tangent of the angle of inclination of the line $\lg\frac{j_{sat}}{T^2}\left(\frac{1}{T}\right)$ plotted from the experimental data:

$$A_{вих} = \frac{k\cdot\tan\alpha}{0.43}, \tag{6.6}$$

where $\tan\alpha = \frac{\Delta(\lg(j_{sat}/T^2))}{\Delta(1/T)}$.

To build the graph, we need to know the saturation current density in the anode and the temperature of the cathode. The cathode temperature is determined by the power of the current, which causes the cathode heating. For the wolfram, this dependence is represented by the graph on Fig. 6.7.

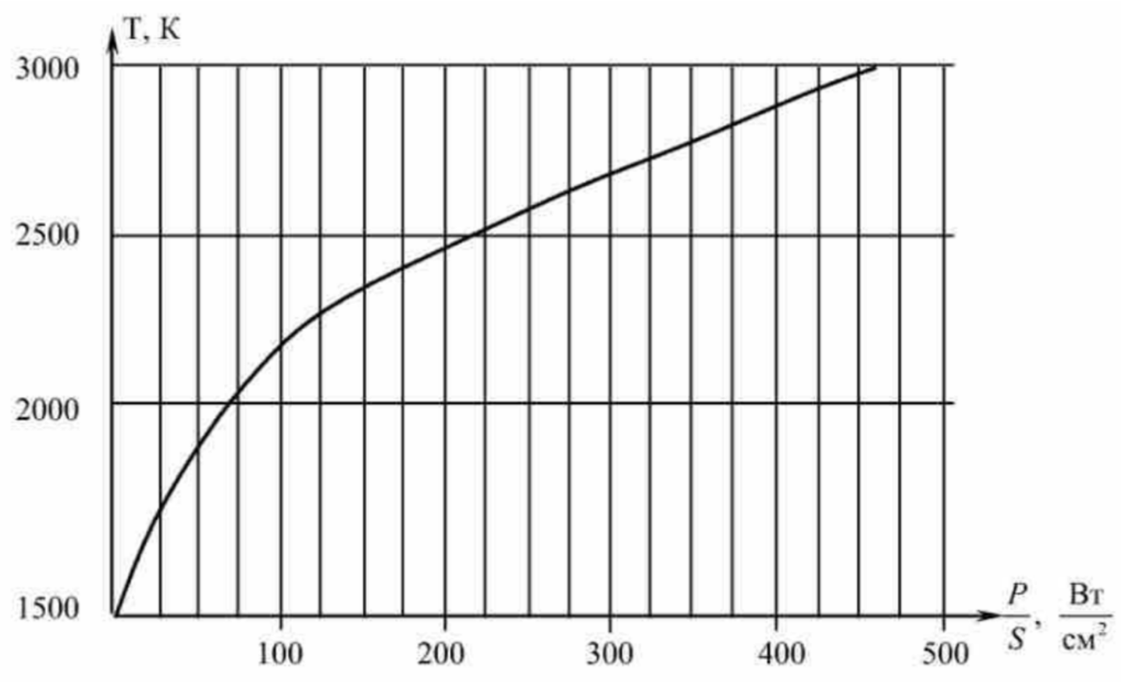



Figure 6.7

The dependence of the cathode temperature on the Joule's power supplied for the cathode heating was determined experimentally. Knowing the power supplied to the cathode per unit area, it is possible to determine its temperature according to the graph (Fig. 6.7).

## Procedure

1. Connect the PPE-06 cassette to the power supply. The ammeter on the panel of the power supply serves to control the heating current $I_h$ supplied to the cathode, whose maximum value should not exceed 2.2 A. The smooth regulation of the heating voltage is carried out with the left handle located under that ammeter. The heating voltage $U_h$ is measured by the individual voltmeter, which is connected to the terminals 2.5 ... 4.5 V of the power supply. The voltmeter on the power supply panel shows the anode voltage $U_a$, which is controlled with the right handle on the panel, located directly under that voltmeter. To measure the anode current $I_a$, a special milliammeter B7-27 measuring currents up to 20 mA is used.
2. Set the heating voltage 3.5 V and, by increasing the anode voltage $U_a$ in step 10 V in the range of 10-120 V, write the corresponding values of the anode current $I_a$ into the Table 1.

3. Perform measurement in accordance with the paragraph 2 for four values of the heating voltage $U_h$ in the range of 3.5 - 4.5 V (3.5, 3.8, 4.2, 4.5 V).

4. For each value of the $U_h$, build a current–voltage characteristic and determine the saturation current $I_{sat}$. (The data obtained in pp. 4-8 write into the Table 2).

5. For each value of the $U_h$, calculate the heating power in the cathode $P_h = I_h U_h$, as well as the power per unit area of the cathode $P_h/ld = I_h U_h / S$.

6. According to the graph of dependence of the cathode temperature $T$ on the $P_h / S$ (Figure 6.7), determine the temperature of the cathode $T$ for each $U_h$.

7. Calculate the saturation current density in the anode $j_{sat} = I_{sat} / S$.

8. Calculate $\lg(j_{sat}/T^2)$.

9. Build a graph of the dependence $\lg(j_{sat}/T^2)$ vs $1/T$.

10. Using the formula (6.6), calculate the work function of the wolfram

$A_{вих} = \dfrac{k \cdot \tan\alpha}{0.43}$, where $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant, $\tan\alpha$ is the angular coefficient (slope) of the straight line on the graph obtained in p.9.

11. Determine the emission constant $B$, whose logarithm is equal to the coordinate of the point of intersection of the graph with the $y$-axis.

12. Calculate the measurement errors.

Parameters of the experimental setup

$S = 11 \cdot 10^{-6}$ m$^2$;   $d = 0.11$ mm $= 1.1 \cdot 10^{-4}$ м;   $l = 32$ mm $= 3.2 \cdot 10^{-2}$ m.

Table 1

| № | $U_h$, V | $I_h$, mA | $U_a$, V | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| | | | $I_a$, mA | | | | | | | | | | | |
| 1 | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | |

Table 2

| № | $U_h$, V | $I_h$, A | $P_h$, W | $P_h / ld$ | $T$, K | $1/T$, $K^{-1}$ | $I_{sat}$, mA | $j_{sat}$, A/m$^2$ | $j_{sat}/T^2$, A/m$^2$K$^2$ | $\lg(j_{sat}/T^2)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |

## Control questions

1. What is the essence of the phenomena of electronic and thermionic emission?
2. What is the work function of the metal?
3. Explain the nature of the forces holding electrons inside the metal.
4. What is the shape of the I–V curve of the vacuum tube diode?
5. What is the saturation current and how does it depend on temperature?
6. Explain the physical nature of the "law of 3/2".
7. What is the Fermi energy?
8. What physical parameters are determined experimentally in this work?
9. What affects the measurement error in this work and how can it be reduced?

# Laboratory work PPE-07

## Study of the hysteresis of ferromagnetic materials

**Objective:** to study experimentally hysteresis of ferromagnetic materials, to calculate and plot the main magnetization curve, to calculate the work of re-magnetization and the coercive force.

### Theoretical information

All materials have magnetic properties. They are determined by the magnitude and orientation of magnetic moments of molecules, ions or atoms composing the material.

The **magnetic moment** of a planar loop enclosing an area $S$ with a current $I$ flowing in it, is defined as

$$\vec{p}_m = IS\vec{n} \qquad (7.1)$$

where $\vec{n}$ is the unit vector normal to the plane of the loop, whose direction is determined with the right screw rule: if the screw rotates with the direction of current, then the direction of its displacement will show the direction of the vector of the positive normal.

In the magnetic field $\vec{B}$, a torque is exerted on the current loop

$$\vec{M} = \left[ \vec{p}_m, \vec{B} \right], \qquad (7.2)$$

the magnitude of which is

$$M = ISB\sin(\vec{p}_m, \vec{B}). \qquad (7.2a)$$

The torque vector tries to rotate the plane of the loop so that directions of the vectors $\vec{p}_m$ and $\vec{B}$ coincide.

The current loop also creates its own magnetic field $\vec{B}_{loop}$. Direction of this field in the center of the loop coincides with the direction of the magnetic moment vector $\vec{p}_m$. In the state of stable equilibrium of the current loop in the magnetic field when

$M = 0$, the resultant magnetic field $\vec{B} = \vec{B}^0 + \vec{B}_{loop}$ at any point on plane inside the loop is always greater than the external magnetic field $\vec{B}^0$.

Magnetization of a material is explained by the fact that its elementary components (atoms, molecules, ions) possess microscopic magnetic moments due to the motion of electrons around the nucleus (orbital magnetic moment $\vec{p}_l$), the intrinsic magnetic moment of the electron (spin magnetic moment $\vec{p}_s$), and the nuclear magnetic moment $\vec{p}_{nuc}$.

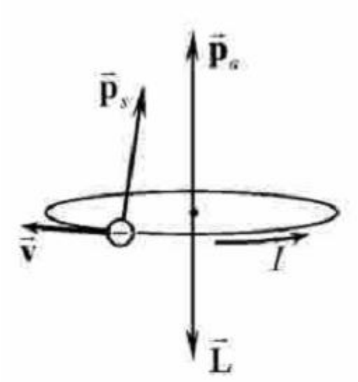



Figure 7.1.

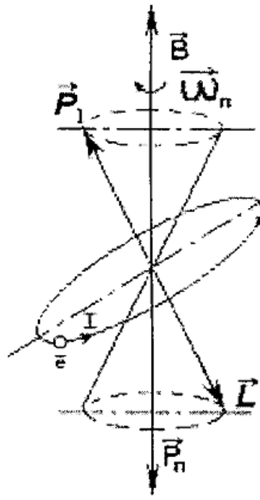



Figure 7.2.

The electronic orbital magnetic moment arises due to the motion of the electron around the nucleus (Fig. 7.1). This motion creates an equivalent current $I = e/T = e\upsilon$ (here $T$ is a period, $\upsilon$ is a frequency of rotation around the nucleus), and is analogous to the circular current loop, whose magnetic moment equals $\vec{p}_l = IS\vec{n} = e\upsilon\pi r^2\vec{n}$, where $e$ is the charge of the electron, $r$ is the radius of its orbit. The direction of the magnetic moment is opposite to the direction of the angular momentum (mechanical momentum) $\vec{L} = [\vec{r}, m\vec{v}]$. The vectors $\vec{p}_l$ and $\vec{L}$ are related with equation $\vec{p}_l = -\frac{e}{2m}\vec{L}$, where $m$ is the mass of the electron.

The spin magnetic moment $\vec{p}_s$ is the inherent quantum property of electron. Spin and orbital magnetic moments are of the order of magnitude of the Bohr magneton

$$\mu_B = \frac{eh}{2m_e} = 9.27 \cdot 10^{-24} A \cdot m^2,$$

where $h = 6.626 \cdot 10^{-34}$ J·s is the Planck constant.

The nuclear magnetic moment $\vec{p}_{nuc}$ is either zero or three to four orders of magnitude lower than $\mu_B$ and its contribution to the resulting magnetic field can be neglected.

In the absence of magnetic field, it can be assumed that the magnetic moment of an atom is the sum of orbital and spin moments of all its electrons

$$\vec{p}_{atom} = \sum_{i=1}^{z} \vec{p}_{li} + \sum_{i=1}^{z} \vec{p}_{si}, \qquad (7.3)$$

where $z$ is the number of electrons in the atom.

Magnetic moment of a molecule consists of magnetic moments of its atoms:

$$\vec{p}_{mol} = \sum_{k=1}^{N} \vec{p}_{atom\ k}$$

where $N$ is the number of atoms in the molecule.

If the atom is placed in the magnetic field, the torque $\vec{M}$ acts on the electrons of the atom (as on the current loops) and causes their precession. That is, under the action of the torque, an electron rotates similarly to a whirligig, so that vectors $\vec{p}_l$ and $\vec{L}$ will rotate with a constant angular velocity about the vector $\vec{B}$, describing a cone (see Fig. 7.2). Such additional motion of the electron creates an equivalent current, and hence a corresponding magnetic moment of precession $\vec{p}_p$, which is directed against the external magnetic field. This phenomenon is called a diamagnetic effect. Thus, in the external magnetic field, the resulting magnetic moment of the atom is

$$\vec{p}_{atom} = \sum_{i=1}^{z} \vec{p}_{li} + \sum_{i=1}^{z} \vec{p}_{si} + \sum_{i=1}^{z} \vec{p}_{pi} \qquad (7.4)$$

Magnetic properties of material are characterized by the **magnetization vector** $\vec{J}$, which is defined as the magnetic moment per unit volume of the magnetic material:

$$\vec{J} = \frac{\sum_{k=1}^{N} \vec{p}_{mol.k}}{\Delta V}, \qquad (7.5)$$

where $\Delta V$ is the elementary volume of the material, $\sum_{k=1}^{N} \vec{p}_{mol.k}$ is the sum of total magnetic moments of all the molecules contained within the volume $\Delta V$.

It is experimentally established that in many materials (diamagnetic and paramagnetic materials) magnetization $\vec{J}$ is directly proportional to the magnetic field strength $\vec{H}$

$$\vec{J} = \chi \vec{H} \tag{7.6}$$

where $\chi$ is the coefficient of proportionality called the **magnetic susceptibilit**y of material.

Magnetic properties of material are also characterized by the relative **magnetic permeability** $\mu$, which is equal to the ratio of the magnetic field in a given material to the magnetic field in vacuum. The values $\chi$ and $\mu$ are related:

$$\mu = 1 + \chi\ . \tag{7.7}$$

Depending on the value of susceptibility, all materials are divided into three groups: diamagnetic, paramagnetic and ferromagnetic materials.

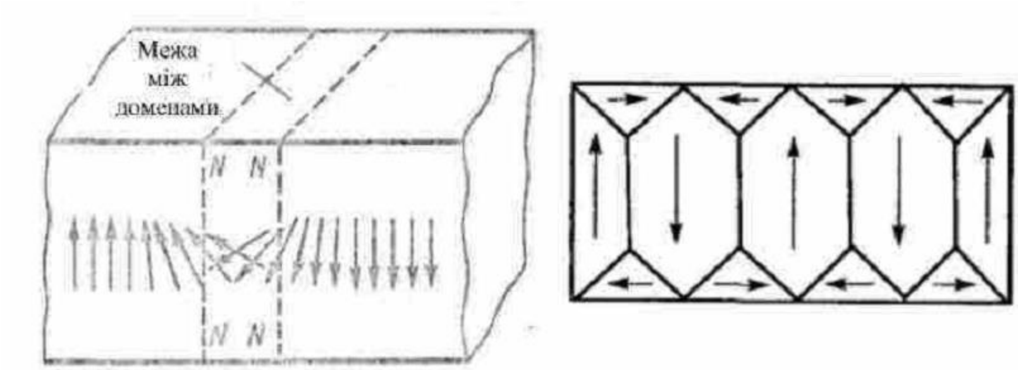
In ***diamagnetic materials*** (for example, inert gas), in the absence of the external magnetic field, orbital $\vec{p}_l$ and spin $\vec{p}_s$ magnetic moments of atoms or molecules are compensated, and the resultant magnetic moment is zero. In the presence of external magnetic field, the induced magnetic moments $\vec{p}_{atom} = \sum_{i=1}^{z} \vec{p}_{pi}$ appear in the result of precession, and are directed against the external field. This reduces the resulting magnetic field in the material, hence the relative magnetic permeability of diamagnetic medium is slightly less than one ($\mu$ < 1), and the magnetic susceptibility is negative and has an order of magnitude $\chi$ ~ $10^{-6}$ - $10^{-8}$.

In ***paramagnetic materials*** in the absence of the external field the resulting magnetic moment of the atom (molecule) is different from zero, $\vec{p}_{atom} \neq 0$ ($\vec{p}_{mol} \neq 0$). In the absence of the external magnetic field, the magnetic moments of the atoms (molecules) have all possible directions due to the thermal chaotic motion, hence the magnetization of the paramagnetic material $\vec{J} = 0$.

But if the paramagnetic material is placed in the external magnetic field, the magnetic moments of atoms start to orientate in the direction of the field (although this is hindered by the thermal chaotic motion), so that the magnetization becomes different from zero. For the paramagnetic materials, the relative magnetic permeability is slightly larger than one ($\mu > 1$), and the magnetic susceptibility is positive, $\chi \sim 10^{-4}$ - $10^{-6}$.

**Ferromagnets** are crystalline materials which have magnetic ordering; that is, their magnetic moments (namely, spin magnetic moments of the neighboring ions in the crystalline lattice) spontaneously align due to the exchange interaction.

Only a few substances are ferromagnetic. The common ones are iron, nickel, cobalt and most of their alloys, and some compounds of rare earth metals. Ferromagnetism is a property not just of the chemical make-up of a material, but of its crystalline structure and microstructure. Ferromagnetic materials have **magnetic domains**, which are the small regions of spontaneous magnetization where individual magnetic moments of the atoms are aligned with one another due to the exchange interaction and they point in the same direction. If all the magnetic moments in a piece of ferromagnetic material are aligned parallel, it creates a large magnetic field containing high magnetostatic energy. The material tends to reduce this energy by splitting into many tiny domains pointing in different directions. That's why ferromagnets are often found in an "unmagnetized" state. Sizes of domains are ~ $10^{-5}$ - $10^{-7}$ m. The regions separating magnetic domains are called domain walls, where the magnetization gradually rotates from the direction in one domain to that in the next domain (Figure 7.3a). Width of the wall is $10^{-8}$ -$10^{-9}$ m.



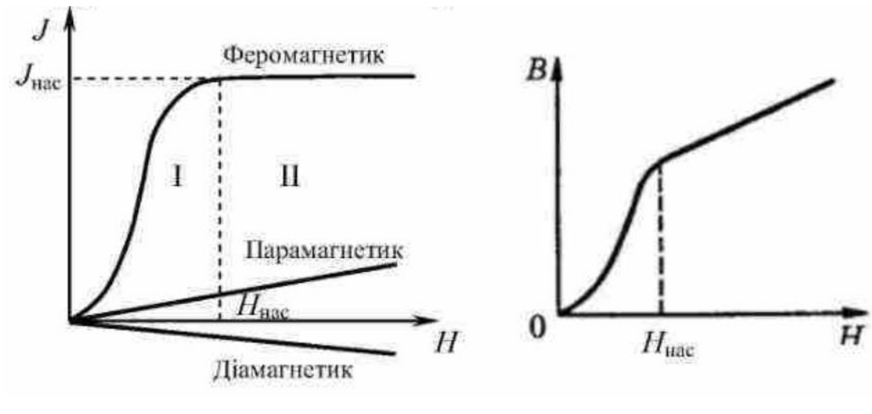

a b

Figure 7.3.

Within each domain, the spins are aligned, but (if the bulk material is in its lowest energy configuration; i.e. unmagnetized), the spins of separate domains point in different directions and their magnetic fields cancel out, so the object has no net large scale magnetic field (Figure 7.3b).

If the ferromagnetic material is placed into the external magnetic field, its domains begin to orientate in the direction of the field and the net magnetization of the material increases, while the transition layers are destroyed.

The dependence of the magnetization *J* on the external magnetic field strength *H* for the dia-, para- and ferromagnetic materials is shown in Fig. 7.4a.



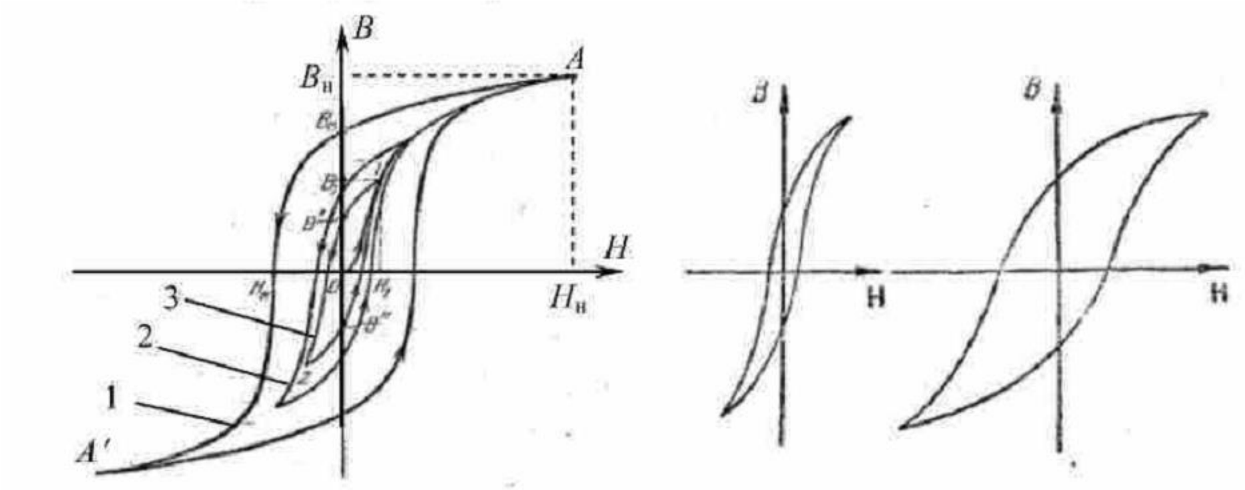

a b

Figure 7.4.

In the region I, orientation of the magnetic domains in the direction of the external field occurs as the field strength *H* increases. The curve *J*(*H*) is called the main magnetization curve. For the para- and diamagnetic materials, the dependence *J*(*H*) is linear.

The main magnetization curve of the ferromagnetic material (the dependence *B*(*H*)) is shown in Fig. 7.4b. Unlike the dependence *J*(*H*), this curve has no saturation.

Ferromagnetic materials and ferrites have **magnetic hysteresis**, which reflects dependence of the magnetization on the previous state. When an external magnetic field is applied to a ferromagnetic material, the magnetic domains align themselves with it. Even when the field is removed, part of the alignment will be retained: the material has become magnetized. Once magnetized, the magnet will stay magnetized indefinitely. To demagnetize it requires heat or a magnetic field in the opposite direction.




a b c

Figure 7.5.

The relationship between the external field strength *H* and the emerged internal field *B* is not linear in such materials. If a magnet is initially demagnetized ($H$=$B$=0), *B* follows the initial magnetization curve OA (see Figure 7.5a). This curve increases rapidly at first and then approaches an asymptote called **magnetic saturation**. At point A, the saturation values $H_{sat}$ and $B_{sat}$ correspond to the state in which all domains are oriented in the direction of the applied magnetic field. If the external magnetic field is now reduced monotonically, *B* follows a different curve ($A$-$B_r$-$H_c$-$A'$). At zero field strength $H = 0$, the magnetization is offset from the origin by an amount called the **remanence** (**residual magnetism**) $B_r$. The strength of the applied magnetic field required to reduce the magnetization of the material to zero ($B = 0$, $J = 0$) is called the **coercivity** (**coercive force**) $H_c$. If the *H*-*B* relationship is plotted for cyclic changes of the applied magnetic field the result is a **hysteresis loop** called the main loop (Fig. 7.5a). The width of the middle section is twice the coercivity of the material.

The curve 1 on the Figure 7.5a is the maximum hysteresis loop when $H_{max} > H_{sat}$. Curves 2 and 3 are partial cycles when $H_{max} < H_{sat}$. The maxima of $B$ and $H$ of the partial cycles lie on the main magnetization curve OA. Ferromagnetic materials with high coercivity $\boldsymbol{H_c}$ > 100 A/m are called magnetically hard materials, and are used to make permanent magnets. Materials with low coercivity $\boldsymbol{H_c}$ < 100 A/m are said to be magnetically soft. Soft ferromagnets have narrow hysteresis loop (Fig. 7.5b), while hard ferromagnets have a nearly rectangular hysteresis loop (Fig. 7.5c).

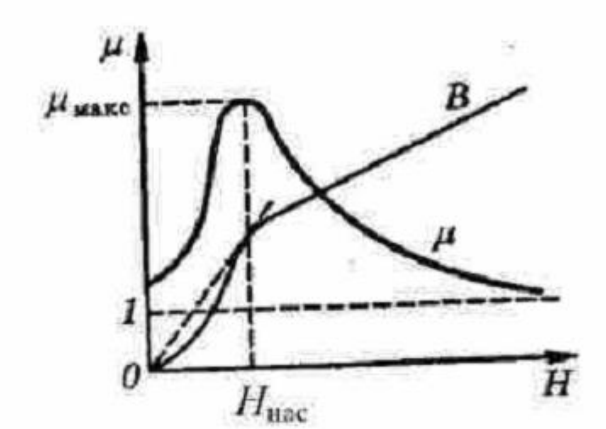



Figure 7.6.

The magnetic permeability $\mu$ of the ferromagnetic material depends on the strength of the applied magnetic field $H$ (Fig. 7.6). It reaches the maximum value when the field strength $H$ equals $H_{sat}$, that is, when domains are maximally aligned in the direction of the applied field, and thus the magnetic saturation of the specimen is reached.

When ferromagnetic materials are heated, their magnetic properties change: values of $\chi$, $\mu$, $J$, etc., are reduced. As the temperature increases, thermal motion result in higher energy electrons disrupting the order and in destruction of the alignment between magnetic moments. When the temperature rises beyond a certain point $T_c$ called the **Curie point** (**Curie temperature**), the material loses its ferromagnetic properties. A phase transition takes place and the ferromagnetic material is converted into a paramagnetic material. For example, Curie points for some ferromagnetic materials are given in Table 7.1.

Table 7.1

| Ferromagnetic material | Iron | Cobalt | Nickel | 30% Permalum |
|---|---|---|---|---|
| $T_c$,°C | 70 | 1150 | 360 | 70 |

Magnetic susceptibility above the Curie temperature can be calculated from the Curie–Weiss law

$$\mu = \frac{C}{T-\theta},$$

where $C$ is Curie constant, $\theta$ is the paramagnetic Curie temperature, which is slightly higher than $T_c$.

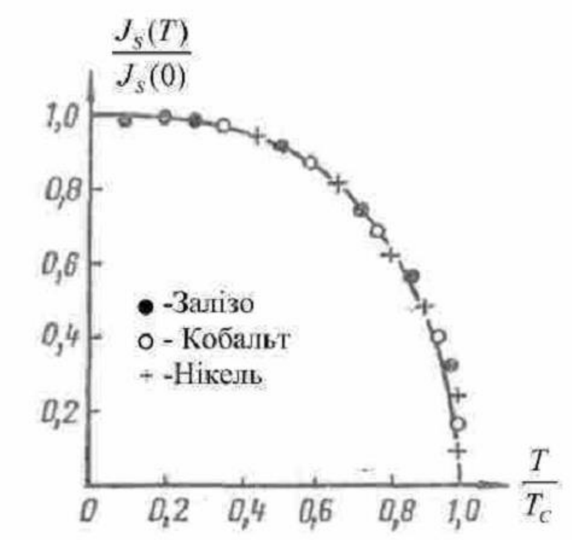



Figure 7.7

In fig. 7.7 shows the dependence of the saturation magnetization of iron, nickel and cobalt on the temperature. At the Curie temperature, the magnetization decreases to zero, and the magnetic permeability decreases by 2-3 orders of magnitude.

**Methodology of measurements and description of the experimental setup**

Present method for studying hysteresis of ferromagnetic materials is based on the use of an oscilloscope. If the voltages $U_y$ and $U_x$, which are proportional to $B$ and $H$ respectively, are applied to the vertical and horizontal deflection plates of the oscilloscope, then the oscilloscope's screen will show a hysteresis loop, the parameters of which are visually recorded and further processed in accordance with instructions given in task 1-3.

The experimental setup includes following devices (Fig. 7.8): sound generator PQ, electronic oscilloscope PO, cassette PPE-07.

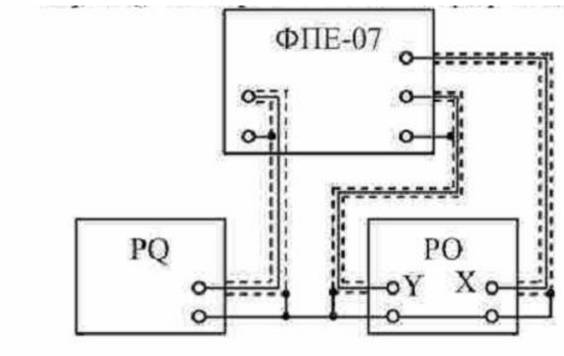



Figure 7.8

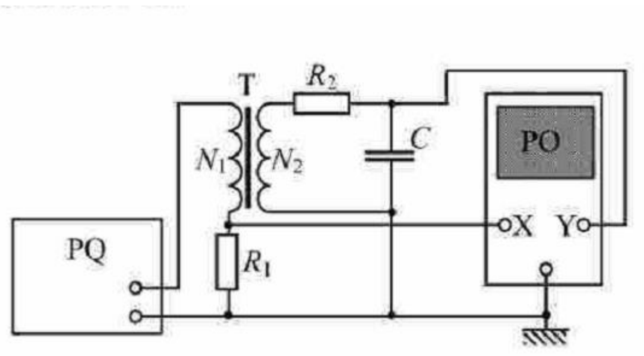



Figure 7.9

The basic circuitry of the experimental setup is shown in Fig. 7.9. The investigated sample is made in the form of a toroidal transformer T, whose primary winding has $N_1$ turns, secondary winding has $N_2$ turns. The voltage from the output of the sound generator PQ is applied to the primary winding of the transformer T through the resistor $R_1$. The secondary winding of the transformer is connected in series with the resistor $R_2$ and the capacitor C. The voltage from the resistor $R_1$ is applied to the input "X" of the oscilloscope’s amplifier of horizontal signal. This voltage $U_x$ is proportional to the magnetic field strength $H$.

The voltage from the capacitor C is applied to the input “Y” of the amplifier of vertical signal. This voltage $U_y$ is proportional to the magnetic field $B$.

If radii of the loops of the winding $r_w < r_t$ ($r_t$ is radius of the middle line of the toroid), then according to the law of circulation of the magnetic field in the toroid,

$$H = \frac{N_1}{2\pi r_t} I_1 \qquad (7.8)$$

The voltage across the resistor $R_1$, and consequently the voltage applied to the input “X” of the oscilloscope, is $U_x = I_1 \cdot R_1$. Expressing $I_1$ from (7.8), we obtain

$$U_x = R_1 \frac{2\pi r_t}{N_1} H \quad . \qquad (7.9)$$

If the coefficient of deflection of the electron beam along the "X" axis is equal to $b_1$, then

$$U_x = b_1 x \quad , \qquad (7.10)$$

where $x$ is the deflection of the electron beam on the oscilloscope’s screen along the X axis. Thus, taking into account (7.10) we can write

$$H = \frac{N_1 b_1}{2\pi r_t R_1} x = \alpha b_1 x \quad . \qquad (7.11)$$

According to the Faraday's law, the induced emf in the secondary winding is

$$\varepsilon_i = -N_2 \frac{d\Phi}{dt} = -N_2 \frac{dB}{dt} S_2 \quad , \qquad (7.12)$$

where $\Phi$ is the magnetic flux through one turn of the winding, $S_2$ is the cross-sectional area of the toroid: $S_2 = (r_2 - r_1) \cdot b_t$, $r_1$, $r_2$ are the outer and inner radii of the toroid, $b_t$ is the height of the toroid.

According to the Kirchhoff’s second rule, the emf in the secondary winding is equal to the sum of the voltage drops across the elements of the circuit of the secondary winding:

$$\varepsilon_2 = U_c + I_2 R_2 + I_2 \frac{dI_2}{dt} \quad , \qquad (7.13)$$

where $U_c$ is voltage across the capacitor, $I_2$ is current in the secondary winding circuit, $L_2$ is inductance of the secondary winding. Since the inductance $L_2$ is very small, and $I_2 \cdot R_2 >> U_c$, then, taking into account (7.12), the expression (7.13) can be rewritten as:

$$-N_2 \frac{dB}{dt} S_2 = I_2 R_2 \quad ,$$

and

$$I_2 = -\frac{N_2 S_2}{R_2}\frac{dB}{dt}. \qquad (7.14)$$

The voltage across the capacitor is $U_c = q/C$, where $q = \int I_2 dt$ is the charge on the capacitor. So, we obtain

$$U_Y = U_c = \frac{\int I_2 dt}{C} = -\frac{N_2 S_2}{R_2 C}\int \frac{dB}{dt}dt = -\frac{N_2 S_2}{R_2 C}B. \qquad (7.15)$$

If the coefficient of deflection of the electron beam along the "Y" axis is $b_2$, then

$$U_y = b_2 y. \qquad (7.16)$$

From the expressions (7.15), (7.16) we obtain

$$|B| = \frac{R_2 C}{N_2 S_2}U_Y = \frac{R_2 C}{N_2 S_2}b_2 y = \beta b_2 y. \qquad (7.17)$$

Applying simultaneously voltages $U_y$ and $U_x$ to vertical and horizontal deflecting plates of the oscilloscope, we obtain the hysteresis loop on the oscilloscope's screen.

Knowing the area of the loop, it is possible to calculate the work of magnetization per unit volume. Small change in the volumetric density of the magnetic field energy

$$d\omega = d(\mu\mu_0 H^2/2) = Hd(\mu\mu_0 H) = HdB \qquad (7.18)$$

in the process of magnetization is equal to the elementary work $\delta A = d\omega$ in changing the internal energy of the unit volume of the ferromagnetic material. During the complete cycle of re-magnetization the total work is

$$A_{re} = \int HdB. \qquad (7.19)$$

Taking into account expressions (7.11) and (7.18), we obtain

$$A_{re} = \frac{N_1 R_2 C b_1 b_2}{2\pi r_t R_1 N_2 S_2}S_{re}, \qquad (7.20)$$

where $S_{re}$ is the hysteresis loop area.

## Procedure

1. Determination of the main curve of magnetization.

1.1. Switch on the laboratory setup and devices. Set the beam in the center of the oscilloscope's screen, and then, by adjusting the value of the output voltage on the sound generator, obtain the maximum hysteresis loop within the screen corresponding to the magnetic saturation of the sample. By reducing the output voltage, observe a family of hysteresis loops at least 8 times. For each loop, write the coordinates X and Y of its vertex into the Table 7.2.

Table 7.2

| Loop number | x, devisions | y, devisions | $U_x=b_1x$, B | $U_y=b_2x$, B | H, A/m | ΔH, A/m | *B*, T | Δ*B*, T |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |
| 8 | | | | | | | | |

1.2. Using the formulas $H = \alpha b_1 x$ and $B = \beta b_2 y$ determine *H* and *B* at the vertices of all the obtained hysteresis loops and write them into the table. The values of the coefficients $b_1$ and $\boldsymbol{b_2}$ are given on the panel of oscilloscope.

Plot a graph of the dependence $B = f(H)$.

1.3. Estimate the error for *H* and *B* measurements with a confidence probability P = 0.9 ($k_p$ = 1.615) due to the errors of the quantities $b_1$, $b_2$, *x, y*

($\Delta b_1 = \Delta b_2 = 0.07$ V/mm, Δx = Δy = 0.5 mm).

Errors of values *B* and *H* are determined by:

a) systematic errors of devices associated with the coefficients of deflection of the electron beam $b_1$ and $b_2$, as well as the visual errors of the *x* and *y* values on the oscilloscope's screen;

b) errors of the given values $N_1$ , $N_2$ , $R_1$ , $R_2$ , $S_2$ , $C$, $r_t$. These elements of the experimental setup are usually made for measuring instruments of high accuracy and do not contribute significantly to the overall error;

c) error associated with some assumptions in the derivation of the calculation formulas (7.11) and (7.17) (systematic methodological error). The confidence intervals of individual measurements in accordance with formula (7.13) and equations (7.11) and (7.17) can be determined with the following relationships

$$\Delta H = a\sqrt{x^2\left(\frac{k_p \Delta b_1}{3}\right)^2 + b_1^2\left(\frac{k_p \Delta x}{3}\right)^2} \qquad (7.21)$$

$$\Delta H = \beta\sqrt{y^2\left(\frac{k_p \Delta b_2}{3}\right)^2 + b_2^2\left(\frac{k_p \Delta y}{3}\right)^2} \qquad (7.22)$$

In these formulas, $\Delta H$ and $\Delta B$ are the confidence intervals for measurement errors of $H$ and $B$.

The values $\pm\,\Delta H$ and $\pm\,\Delta B$ should be plotted on the curve $B(H)$, and also should be written into the table 7.2.

2. Estimation of the re-magnetization work $A_{re}$ during one cycle.

2.1. Obtain the maximum hysteresis loop and draw it on transparent paper in coordinates $x$ and $y$.

2.2. Copy the loop on the millimeter paper and calculate its area.

2.3. Using the formula (7.20), determine the re-magnetization work $A_{re}$ during one cycle.

3. Determination of coercive force.

3.1. On the maximum hysteresis loop, determine the coordinate $x_c$, which corresponds to the coercive force.

3.2. Calculate the value of $H_c$ using the formula (7.11).

3.3. Determine if the ferromagnetic material is soft or hard.

Data for the laboratory setup:

$$N_1 = 200\ turns,\ \ N_2 = 75\ turns$$
$$R_1 = 400\Omega,\ \ R_2 = 24k\Omega$$
$$C = 0.022\mu F$$

Toroid parameters:

$$d_1 = 2r_1 = 31mm$$
$$d_2 = 2r_2 = 18.5mm$$
$$b_t = 7mm$$
$$r_t = \frac{d_1 + d_2}{4}$$

Formulas for calculation:

$$\alpha = \frac{N_1}{2\pi R_1 r_t}$$

$$\beta = \frac{R_2 C}{N_2 S_2}$$

$$A_{re} = \frac{N_1 R_2 C b_1 b_2}{2\pi r_t R_1 N_2 S_2} S_{re}$$

## Control questions

1 What is the magnetic field? What are the magnetic field vectors $\vec{B}$ and $\vec{H}$, and what is the relation between them?

2. What happens during magnetization of a magnetic material? What is the physical meaning of the magnetization vector?

3. What types of magnetic materials exist? What are the properties of the dia- and paramagnetic materials?

4. What are the properties of the ferromagnetic materials? What is the phenomenon of magnetic hysteresis?

5. What is the nature of ferromagnetism? What is the Curie point for a ferromagnetic material?
6. What is the phenomenon of electromagnetic induction? Formulate the Faraday's law for the electromagnetic induction.
7. Formulate the Ohm's law for alternating current.
8. Describe the principal diagram of the experimental setup for the study of hysteresis. What is the principle of its operation?

## Laboratory work PPE-10

## Investigation of damped oscillations in the oscillating circuit

**Objective:** to determine parameters and characteristics of a real oscillating circuit.

**Equipment:** a sound generator ГЗ-111, an oscilloscope С1-76, a cassette with a circuit ФПЕ -10/11, pulse converter ПИ/ФПЕ -09, a power supply ИП, a rheostat.

The diagram of the experimental setup is shown in Fig. 10.1

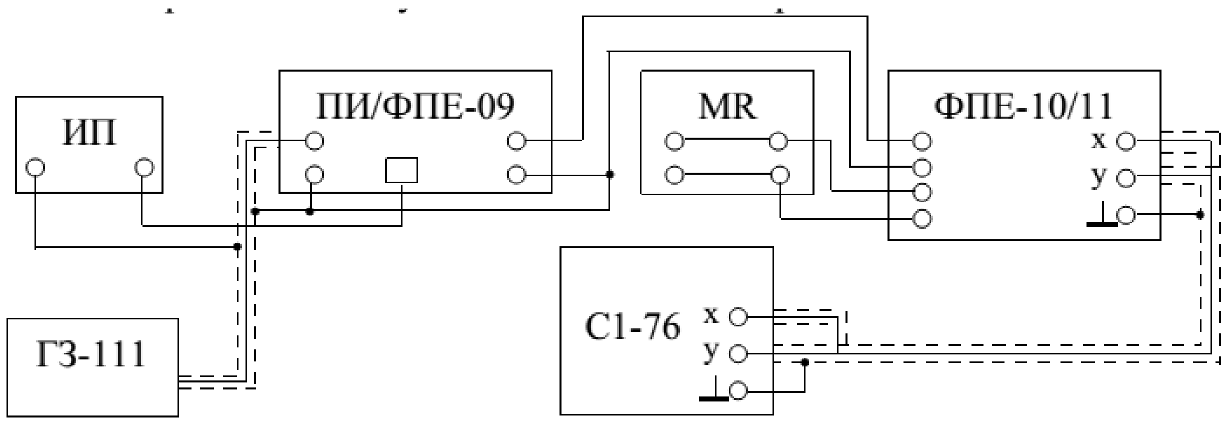
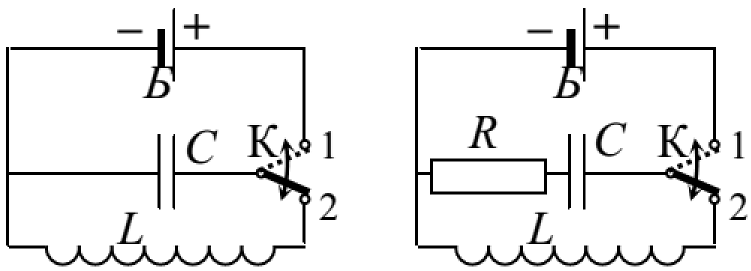



Figure 10.1

### Theoretical information

If one charges a capacitor from the battery to the potential difference *U* (Fig. 10.2a), and then switches the key K from position 1 to position 2, then the capacitor *C* will start to discharge through the solenoid *L*, and electromagnetic oscillations will occur in the circuit.




a b

Figure 10.2

First, let us consider the case when the circuit resistance $R = 0$. After the circuit is closed (the key in position 2), a current *i* starts to flow in it. A change in the current in the circuit causes appearance of the self-induced emf $\varepsilon_s$ in the solenoid, which, according to the Lenz's rule, prevents current changes. That is, it slows down the discharge of the capacitor. But when the current in the circuit decreases, the self-induced emf supports the current that caused its appearance. That leads to the recharge of the capacitor, and then the process is repeated, but with opposite current direction. Subsequently, these processes are repeated, that is, electromagnetic oscillations occur.

The time, during which the capacitor is completely discharged and then recharged, is called the period of natural oscillation.

At the initial moment, when the capacitor is charged, it has accumulated energy

$$W_e = \frac{CU^2}{2}.$$

During discharge, the electric field energy stored in the capacitor is converted into the magnetic field energy of the solenoid, and when the capacitor is completely discharged, the magnetic field energy reaches the maximum value

$$W_m = \frac{LI_0^2}{2},$$

where $I_0$ is the amplitude of the current in the circuit, $L$ is inductance of the solenoid. When the capacitor is recharged, the magnetic field energy is transformed again into the electric field energy. So, continuous harmonic electromagnetic oscillations occur in the circuit.

All without exception, conductors under normal conditions have non-zero resistance, so during oscillations part of the energy is spent on the resistors' heating, that is, it is converted into the heat and lost. As a result, the amplitude of electromagnetic oscillations in the circuit decays and oscillations are damping (Fig. 10.3). Such an oscillatory circuit is called the RLC circuit (resistor-inductor-capacitor).

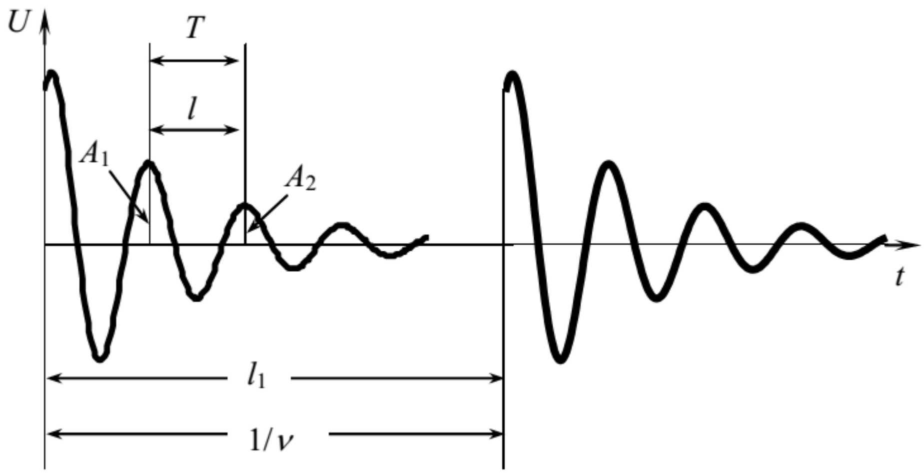



Figure 10.3

At a sufficiently large circuit resistance or small inductance, oscillations do not arise at all, and so-called aperiodic discharge of the capacitor takes place.

In the RLC circuit the self-induced emf $\varepsilon_s$ arises in the inductor (solenoid), so for the real oscillatory circuit (Fig. 10.2b), according to the second Kirchhoff's rule, we can write

$$Ri + U_c = \varepsilon_s , \tag{10.1}$$

where $Ri$ is the voltage drop across the resistor, $U_c$ is the voltage across the capacitor.

Taking into account that $i = \dfrac{dq}{dt}$ and $U_c = \dfrac{q}{C}$ and $\varepsilon_s = -L\dfrac{di}{dt} = -L\dfrac{d^2q}{dt^2}$, we can write

$$R\frac{dq}{dt} + \frac{q}{C} = -L\frac{d^2q}{dt^2} . \tag{10.2}$$

Now let us substitute $q = CU$

$$RC\frac{dU}{dt} + U = -LC\frac{d^2U}{dt^2} . \tag{10.3}$$

In the canonical form, this equation has the form:

$$\frac{d^2U}{dt^2} + \frac{R}{L}\frac{dU}{dt} + \frac{1}{LC}U = 0 . \tag{10.4}$$

Let us introduce notations:

$$\beta = \frac{R}{2L} \tag{10.5}$$

is the damping coefficient,

$$\omega_0^2 = \frac{1}{LC} \tag{10.6}$$

is the natural frequency. Now the equation (10.4) takes the form

$$\frac{d^2U}{dt^2} + 2\beta\frac{dU}{dt} + \omega_0^2 U = 0 \tag{10.4a}$$

If $\omega_0^2 > \beta^2$, then solution of the equation (10.4a) is a function

$$U = U_0 e^{-\beta t}\cos(\omega t + a), \tag{10.7}$$

where $\omega$ is the angular frequency of the damped oscillations:

$$\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \tag{10.8a}$$

In such a case, period of the damped oscillations is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}. \tag{10.8b}$$

If we multiply (10.4a) by $C$ (in such a way we obtain the charge of the capacitor), consider that $\frac{dq}{dt} = i$ and then take the time derivative of the equation, we obtain the equation

$$\frac{d^2i}{dt^2} + 2\beta\frac{di}{dt} + \omega_0^2 i = 0 \tag{10.4b}$$

The solution of (10.4b) is function $i(t)$ which is similar to (10.7), i.e. the current in the circuit undergoes damped oscillations, however, the initial phase of those oscillations is different.

From (10.8a) and (10.8b) we see that the damped oscillations in the circuit are possible only when $(R/2L)^2 < 1/LC$ (frequency and period are real values), or $R < 2\sqrt{\frac{L}{C}}$. If $R > 2\sqrt{\frac{L}{C}}$, there is no oscillation in the circuit, but there is aperiodic discharge of the capacitor. Resistance

$$R_{cr} = 2\sqrt{\frac{L}{C}} \qquad (10.9)$$

is called critical.

In order to characterize the damped oscillations, in addition to the damping coefficient $\beta$, a logarithmic decrement $\lambda$ is used. The logarithmic decrement of damping is the natural logarithm of the ratio of voltage values which are separated by a time interval equal to the period of oscillations $T$,

$$\lambda = \ln\frac{A_1}{A_2} = \ln\frac{A(t)}{A(t+T)}, \qquad (10.10)$$

or

$$\lambda = 2.3\cdot\lg\frac{A_1}{A_2} = 2.3\cdot\lg\frac{A(t)}{A(t+T)}. \qquad (10.10a)$$

If we substitute expression for the damped oscillations amplitude $A(t) = U_0 e^{-\beta t}$ and $A(t+T) = U_0 e^{-\beta(t+T)}$ into (10.10), we obtain

$$\lambda = \beta T, \qquad (10.11)$$

or taking into account (10.8b)

$$\lambda = \beta T = \frac{R}{2L}\frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}. \qquad (10.11a)$$

In some cases, it is convenient to study the oscillatory process in the $i$ - $U$ coordinate system, that is, to plot values of the current in the circuit on the abscissa axis and values of the voltage across the capacitor at the same instant of time on the ordinate axis. The $i$ - $U$ plane is called the plane of states, or the phase plane, and the curve depicting the dependence of current on voltage is called the phase curve (Fig. 10.4).

Let us obtain the phase curve for the circuit with resistance $R = 0$. In such a case, $\beta = R/2L = 0$ and then (10.7), (10.8a) and (10.8b) transform into $\omega = \frac{1}{\sqrt{LC}}$;

$T = 2\pi\sqrt{LC}$, and

$$U = U_0 \cos \omega t . \qquad (10.12)$$

The current in the circuit

$$i = -C\frac{dU}{dt} = U_0 \omega \sin \omega t . \qquad (10.13)$$

Equations (10.12), (10.13) describe harmonic oscillations. Excluding the time $t$ from them, we obtain equation of the phase curve (equation of the ellipse):

$$\frac{U^2}{U_0^2} + \frac{i^2}{(U_0 \omega C)^2} = 1. \qquad (10.14)$$

The ellipse can be obtained as a result of overlapping of two mutually perpendicular harmonic oscillations (10.12), (10.13) with the phase shift of the quarter of period.

In the circuit with resistance $R > 0$, there are damped oscillations of voltage (10.7) and current:

$$U = U_0 e^{-\beta t} \cos \omega t;$$
$$i = -C\frac{dU}{dt} = U_0 C e^{-\beta t}(\beta \cos \omega t + \omega \sin \omega t). \qquad (10.15)$$

In such a case, the amplitudes of voltage and current in the circuit continuously decrease and the phase curve will be unclosed (Fig. 10.4).

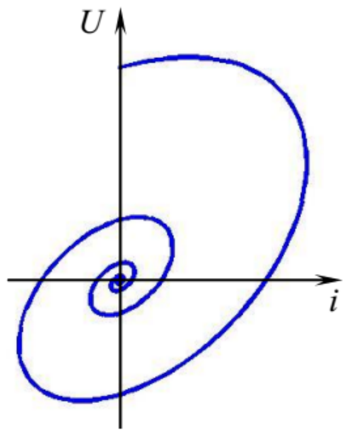



Figure 10.4

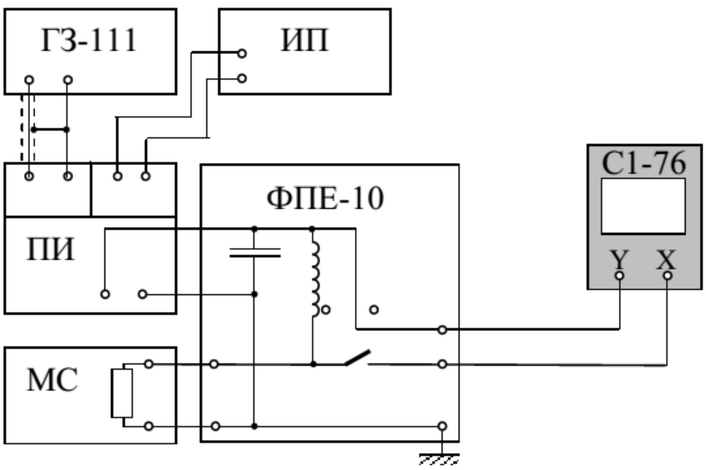



Figure 10.5

In this work, the ФПЕ-10/11 cassette is used to obtain oscillations in the circuit. The circuit diagram is shown in Fig. 10. 5. Damped oscillations that occur in the circuit are observed on the screen of the oscilloscope C1-76. The charging and discharging cycle of the capacitor lasts during time $1/\upsilon$, where $\upsilon$ is the frequency generated by the sound generator ГЗ-111. A segment $l_1$ on the oscilloscope screen corresponds to that time. A segment $l$ corresponds to the period $T$ of the damped oscillations, and it allows us to determine it (see in Fig. 10.3). From the proportion $(l/T) = l_1\upsilon$ we obtain:

$$T = \frac{l}{l_1\upsilon}. \tag{10.16}$$

**Procedure**

TASKS 1. Measurement of the period, logarithmic decrement of damping and parameters $R$, $L$, $C$ of the oscillatory circuit.

Laboratory equipment is shown in Fig. 10.6.

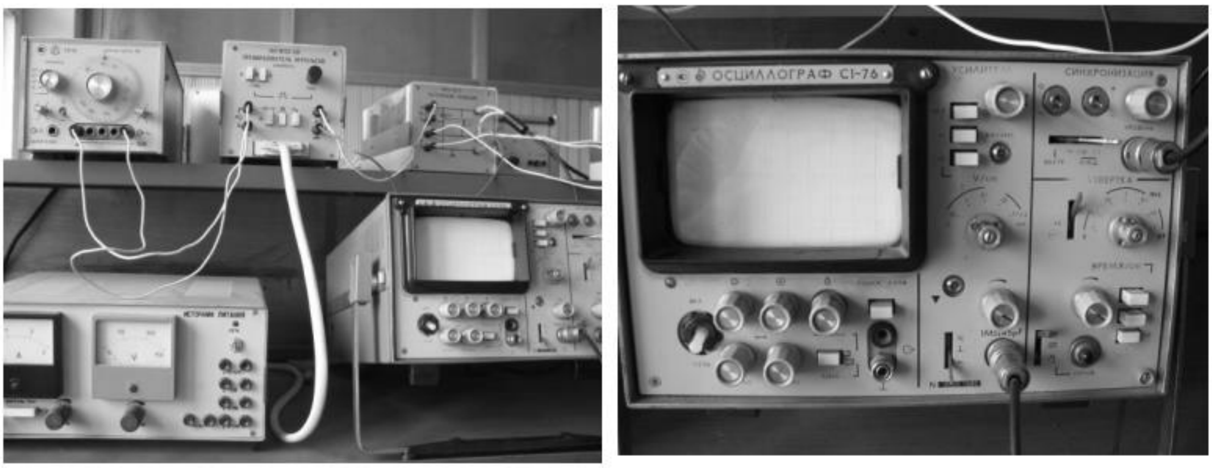



Figure 10.6

1. Switch on the laboratory setup.

2. Turn on the sound generator ГЗ-111. Set the output frequency $\upsilon$ = 250 Hz.

3. Turn on the power supply by pressing the "Power" button.

4. On the pulse converter "ПИ/ФПЕ-09" press the button "П" and the right button for duty cycle "Скважність грубо".

5. Set the resistance $R_m$ = 100 Ω with the rheostat.

6. Turn on the oscilloscope C1-76 with the tumbler "Power".

7. Use the "Focus", "Brightness", "Stable", "Level" knobs to get a steady picture of the oscillations on the screen.

8. Measure the distances $l_1$ and $l$ and calculate the oscillations period $T$.

9. Measure the amplitudes of oscillations $A_1$, $A_2$, $A_3$ and, by combining them in pairs, calculate the logarithmic decrement of damping. Calculate the damping coefficient. Write the results of measurements and calculations in Table 1.

10. Repeat measurement from p.9 setting the values of $R_m$ = 300, 500, 600 Ω with the rheostat.

Table 1

| $R_m$ | $A_1$ | $A_2$ | $A_3$ | $\lambda$ | $\beta$ | $L$ | $C$ | $r_к$ | $R$ |
|---|---|---|---|---|---|---|---|---|---|
| 100 | | | | | | | | | |
| 300 | | | | | | | | | |
| 500 | | | | | | | | | |
| 600 | | | | | | | | | |

Build a graph of the dependence of the logarithmic decrement of damping on the rheostat resistance $R_m$, plotting the values of $R_m$ on the abscissa axis starting from an arbitrary starting point and extrapolating the graph to $\lambda = 0$. The total resistance of the circuit $R$ consists of the rheostat resistance $R_m$ and $r_k$:

$$R = r_k + R_m,$$

where $r_k$ corresponds to the point of intersection of the straight line with the abscissa axis (Fig. 10.6). According to the formula (10.11a)

$$\lambda = \frac{r_k + R_m}{2L} T \qquad (10.17)$$

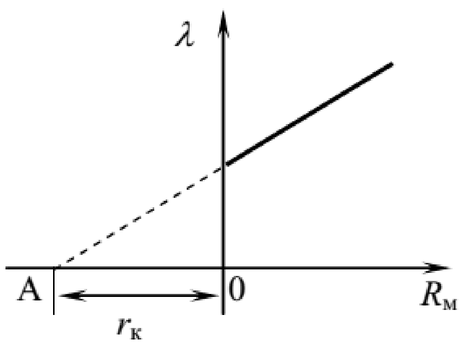



Figure 10.6

11. Using the obtained value of $r_k$ and the value of the period $T$, calculate the inductance and capacitance.

12. Find the rheostat resistance $R_{m\_cr}$ at which the aperiodic discharge of the capacitor takes place. Calculate $R_m$ with the formula:

$$r_k + R_{m\_cr} = 2\sqrt{\frac{L}{C}}.$$

TASKS 2. Investigation of the phase curves

In order to observe the phase curve on the oscilloscope's screen, one should supply voltage from the plates of the capacitor to the vertically-deflecting plates, while the voltage from the terminal of the rheostat $R_m$, which is proportional to the current $U_R = i\ R_{\mathrm{m}}$, should be supplied to the horizontally-deflecting plates. Thus, the dependence of the voltage $U$ across the plates of the capacitor on the current $i$ flowing in the circuit is observed on the oscilloscope's screen.

1. Switch the oscilloscope's tumbler "Sweep" down to the end.
2. Use the "↔" and "↕" knobs to set the picture in the center of the screen.
3. By changing the rheostat resistance $R_m$, obtain the phase curves for different resistance values.
4. Measure the voltage values separated by time of the oscillations period, that is, measure distances from the origin of the phase curve to the points of intersection of the spiral turns with the voltage axis $U$. Use them to calculate the logarithmic decrement of damping:

$$\lambda = \ln\frac{U_1}{U_2}$$

Similarly, calculate the logarithmic decrement using the values of current $i$ separated by the time of the period:

$$\lambda = \ln\frac{i_1}{i_2}$$

5. Write the results into Table 2:

Table 2

| $R_m$ | $R_m + r_k$ | $U_1$ | $U_2$ | $U_3$ | $\lambda$ | $i_1$ | $i_2$ | $i_3$ | $\lambda$ |
|---|---|---|---|---|---|---|---|---|---|
| 100 | | | | | | | | | |
| 300 | | | | | | | | | |
| 500 | | | | | | | | | |
| 600 | | | | | | | | | |

6. Make measurements according to p.4 for the rheostat resistance values 100, 300, 500, 600 Ω.

7. Calculate the measurement error

$$\Delta\lambda = \sqrt{\frac{\Delta U_1^2}{U_1^2} + \frac{\Delta U_2^2}{U_2^2}},$$

where $\Delta U$ is the measurement error on the screen.

8. Draw the phase curve of harmonic oscillations in the circuit.

## Control questions

1. How do oscillations occur in the oscillating circuit?
2. Derive the equation of oscillations for the RLC oscillatory circuit.
3. What kind of solution does the derived equation of the oscillatory circuit have?
4. What formula describes time dependences of the voltage across the capacitor, electric current, electrical and magnetic energies in the oscillatory circuit?
5. What is the damping time and the logarithmic decrement of damping?
6. How does the logarithmic decrement of damping depend on the ohmic resistance of the circuit?
7. What is the aperiodic discharge of the capacitor and what are conditions for it?
8. What is the phase plane and the phase curve?
9. What is the shape of the phase curve for the case of harmonic oscillations? For the case of damped oscillations? For the case of the aperiodic process?

10. From which circuit elements should the voltage be supplied to the oscilloscope's deflecting plates in order to observe the damped oscillations? To observe the phase curve?

11. Explain processes that take place in the oscillatory circuit at the moments when the phase curve intersects voltage axis or current axis.

12. Under what conditions can harmonic oscillations be obtained in the given circuit?

## Laboratory work № PPE-11

## Investigation of driven oscillations in the oscillating circuit

**Objective:** to study the resonance phenomenon in the series RLC circuit

**Equipment:** a sound generator ГЗ-111 (PQ), an oscilloscope C1-76 (PO), a cassette with a circuit ФПЕ - 11, a rheostat (MR), a set of capacitors (MC).

The diagram of the experimental setup is shown in Fig. 11.1.

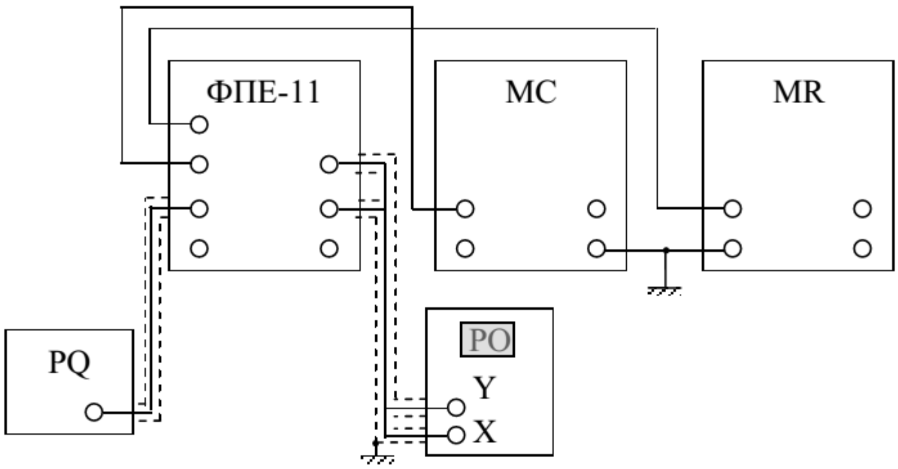



Figure 11.1

### Theoretical information

Consider processes that take place in a series oscillating circuit connected to a source of electromotive force which is varying according to the harmonic law:

$$\varepsilon = \varepsilon_0 cos\Omega t \tag{11.1}$$

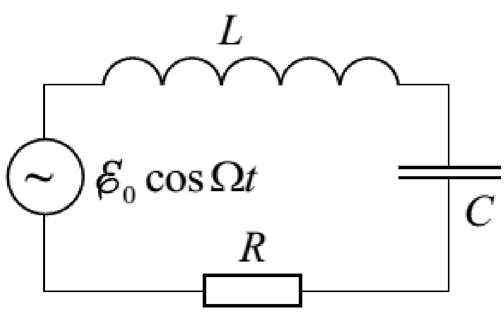



Figure 11.2

Let us denote that $U$ is the voltage across a capacitor of capacitance $C$, $U_L$ is the voltage drop across a solenoid of inductance $L$, and $I$ is the current in the circuit. If the current is considered stationary, then the current and voltage in the conductor are subject to the same laws as the direct current. According to the Kirchhoff's second rule, the sum of voltage drops across the circuit elements is equal to the sum of electromotive forces acting in that circuit (Fig. 11.2). Thus, we can write:

$$U_L + IR + U = \varepsilon_0 \cos \Omega t \,. \tag{11.2}$$

The voltage across the solenoid is numerically equal to the self-induced emf

$$U_L = -\varepsilon_s = L\frac{dI}{dt} \,. \tag{11.3}$$

Current in the circuit causes change of charge on the capacitor, therefore

$$I = \frac{dq}{dt} = \frac{d(CU)}{dt} = C\frac{dU}{dt} \,. \tag{11.4}$$

Substituting (11.3) into (11.4), and then substituting the obtained expressions into (11.2), we obtain

$$LC\frac{d^2U}{dt^2} + RC\frac{dU}{dt} + U = \varepsilon_0 \cos \Omega t \,. \tag{11.5}$$

Let us represent the last equation in the canonical form, dividing all its parts by *LC*:

$$\frac{d^2U}{dt^2} + \frac{R}{L}\frac{dU}{dt} + \frac{1}{LC}U = \frac{\varepsilon_0}{LC}\cos \Omega t \,. \tag{11.6}$$

Let us introduce notations $R/2L = \beta$, $\omega_0 = 1/\sqrt{LC}$. Then,

$$\frac{d^2U}{dt^2} + 2\beta\frac{dU}{dt} + \omega_0^2 U = \omega_0^2 \varepsilon_0 \cos \Omega t \,.$$

Solution of the nonhomogeneous differential equation of the second order (11.6) is equal to the sum of complete solution of the corresponding homogeneous equation and partial solution of the inhomogeneous equation. Provided that $\omega_0^2 > \beta^2$, the solution of the equation

$$\frac{d^2U}{dt^2}+2\beta\frac{dU}{dt}+\omega_0^2U=0 \qquad (11.7)$$

is the function

$$U_1=U_{01}e^{-\beta t}\cos(\omega t). \qquad (11.8)$$

This equation is the equation of damped oscillation. Damping is determined by the factor $e^{-\beta t}$. In time $\tau = 1/\beta$, called the relaxation time, amplitude of the oscillations decreases $e$ times. (Damping of oscillations in the circuit is due to the heating of conductors, that is, due to conversion of the energy of electric and magnetic fields into thermal (internal) energy). The component $U_1$ is significant when $t \leq \tau$, that is, it determines the transient process when the oscillations are establishing. If $t >> \tau$, this component in the general solution can be neglected.

Under the action of the source of variable emf, oscillations with the frequency of that emf are established in the circuit, but with a phase shift $\varphi$:

$$U=U_0\cos(\Omega t-\varphi) \qquad (11.9)$$

with

$$U_0=\frac{\varepsilon_0\omega_0^2}{\sqrt{(\omega_0^2-\Omega^2)+4\beta^2\Omega^2}}; \qquad (11.10)$$

$$tg\varphi=-\frac{2\beta\Omega}{\omega_0^2-\Omega^2}. \qquad (11.11)$$

Current in the circuit $I=C\frac{dU}{dt}=-\Omega CU_0\sin(\Omega t-\varphi)=I_0\cos(\Omega t-\psi)$, where $\psi = \varphi + \pi/2$. The amplitude of the current in the circuit, as well as the voltage across the capacitor, depends on the relation of frequencies $\Omega$ and $\omega_0$:

$$I_0=\frac{\varepsilon_0C\omega_0^2\Omega}{\sqrt{(\omega_0^2-\Omega^2)+4\beta^2\Omega^2}}. \qquad (11.12)$$

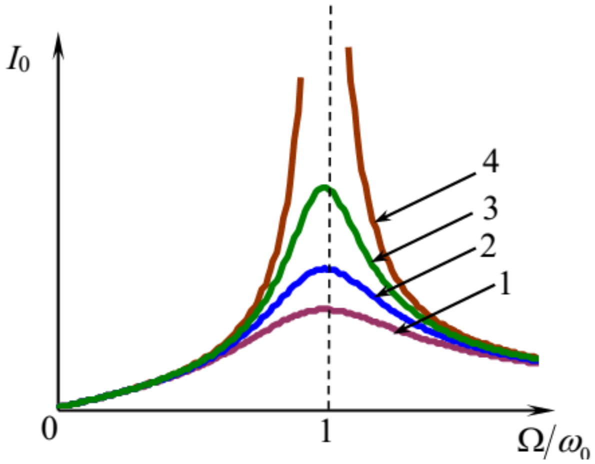



Figure 11.3

Graph of the dependence of $I_0$ on $\Omega/\omega_0$ is shown in Fig. 11.3. It displays that the current amplitude increases sharply when the angular frequency $\Omega$ of the source of emf is close to the natural frequency of the circuit $\omega_0$. This phenomenon is called resonance in the electric circuit, and such curves are called the resonance curves. The value of the current maximum depends on $\beta$, and at $\beta = 0$ the amplitude $I_0 \to \infty$ (curve 4); with increasing $\beta$ the maximum value of $I_0$ decreases (curves 3, 2 and 1).

A phase shift $\psi$ between the current oscillations in the circuit and external emf is

$$tg\psi = tg(\frac{\pi}{2} + \varphi) = -\frac{1}{tg\varphi} = \frac{\omega_0^2\Omega^2}{2\beta\Omega}. \qquad (11.13)$$

Graph of the dependence of $\psi$ on the frequency $\Omega$ is shown in Fig. 11.4. Curves 1 and 2 correspond to different values of $\beta$. For $\Omega = \omega_0$ $tg\psi = 0$ and $\psi = 0$.

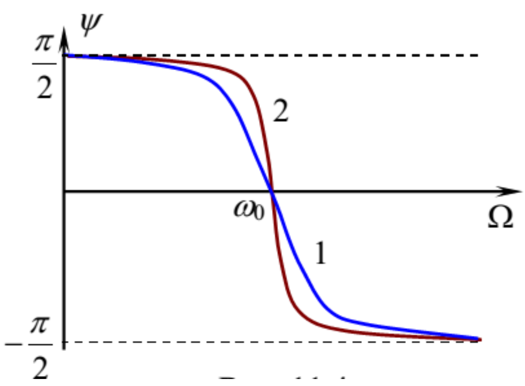



Figure 11.4

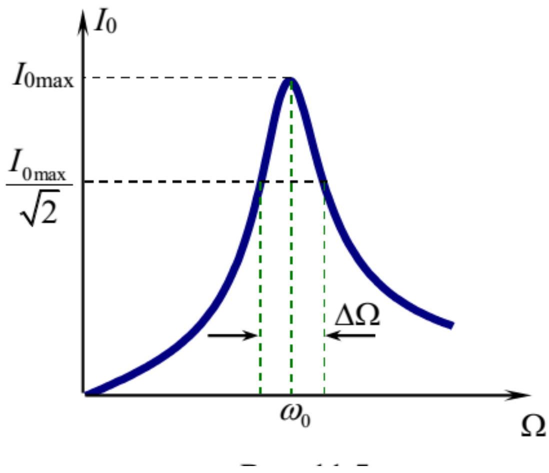



Figure 11.5

The value $Q = \pi / \beta T = \omega / 2\beta$ (here $T$ is the period of damped oscillations, $\omega$ is their angular frequency) is called the Q-factor of the oscillating circuit.

The Q-factor of the circuit determines the sharpness of the resonance curves. Let us find the width of the resonance curve $\Delta\Omega$ at the level $I_0 = \frac{I_{0\max}}{\sqrt{2}} = 0.707 \cdot I_{0\max}$ (Fig. 11.5). From the formula (11.12) it follows that the maximum value of the current amplitude $I_{0\max} = (\varepsilon_0 C \omega_0^2) / 2\beta$, and

$$I_0 = \frac{2\beta\Omega I_{0\max}}{\sqrt{(\omega_0^2 - \Omega^2) + 4\beta^2\Omega^2}}. \qquad (11.14)$$

If $I_0 = I_{0\max} / \sqrt{2}$, we can write (11.4) as

$$\frac{1}{\sqrt{2}} = \frac{2\beta\Omega}{\sqrt{(\omega_0^2 - \Omega^2) + 4\beta^2\Omega^2}}. \qquad (11.15)$$

Expression (11.15) can be reduced to the form $2\beta\Omega = \omega_0^2 - \Omega^2$, or $2\beta\Omega = (\omega_0 - \Omega)(\omega_0 + \Omega)$. The value $\omega_0 - \Omega = \Delta\Omega / 2$, and near the resonance $\omega_0 \approx \Omega$. After the substitution we obtain that $\Delta\Omega = 2\beta$ and

$$\frac{\Delta\Omega}{\omega_0} = \frac{2\beta}{\omega_0} \approx \frac{1}{Q}. \qquad (11.16)$$

If damping is small $(\beta << \omega_0)$, we have $\omega \approx \omega_0$, and the relative width of the resonance curve is numerically equal to the value which is inverse of the quality factor of the circuit. If we know parameters of the oscillating circuit, then the Q-factor can be calculated from the relation

$$Q = \frac{\omega_0}{2\beta} = \frac{1}{R}\sqrt{\frac{L}{C}}. \qquad (11.16a)$$

The principal electric diagram of the experimental setup is shown in Fig. 11.6.

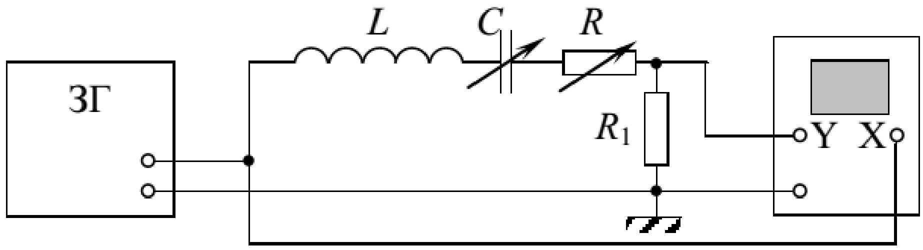



Figure 11.6

The oscillating circuit consists of a solenoid $L$, a set of capacitors $C$, a variable resistor $R$ and a resistor $R_1$. The voltage across the resistor $R_1$, which is proportional to the current in the circuit, is applied to the input "Y" of the electronic oscilloscope, and the sound generator signal is applied to the input "X". To obtain the resonance curves, one should measure dependence of the current in the circuit on the frequency of the generator $I_0$ (Ω) while changing the frequency of the sound generator ЗГ at different values of resistance $R$.

To measure the phase shift $\psi$, the Lissajous figures observed on the oscilloscope screen are used. Suppose there are two sinusoidal voltages of the same frequency Ω. If they are applied to the vertical and horizontal deflecting plates of the oscilloscope, then corresponding displacements of the electronic beam on the screen take place:

horizontal $x = x_0 \cos \Omega t$,

vertical $y = y_0 \cos(\Omega t + \delta)$,

where $\delta$ is the phase shift between the voltages; $x_0$ and $y_0$ are the beam displacement amplitudes that are proportional to the voltage amplitudes and the amplification coefficients of the corresponding oscilloscope channels. Excluding the time from the last two equations, we obtain

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 - \frac{2xy}{x_0 y_0}\cos\delta = \sin^2\delta . \qquad (11.17)$$

Expression (11.17) is equation of an ellipse, which is created by the electron beam on the oscilloscope screen. If the amplification coefficients are chosen so that the equality $x_0 = y_0$ is satisfied, then equation (11.17) takes form

$$x^2 + y^2 - 2xy\cos\delta = x_0^2 \sin^2\delta \qquad (11.18)$$

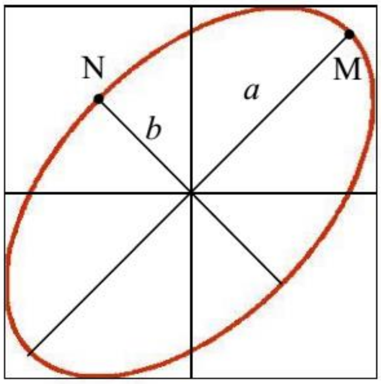



Figure 11.7

Formula (11.18) is equation of the ellipse whose axes form angles π/4 with the coordinate axes. When $\delta = 0$ the ellipse degenerates into a straight line $x = y$, and when $\delta = \pi/2$ it turns into circle of radius $x_0 = y_0$. For the point M on the ellipse (see Figure 11.17) $x = y$, $a^2 = x^2 + y^2 = 2x^2$ therefore, the equation (11.18) for this point is

$$2x^2 - 2x^2\cos\delta = x_0^2\sin^2\delta; \quad 2x^2(1-\cos\delta) = x_0^2\sin^2\delta;$$

$$2x^2 2\sin^2\frac{\delta}{2} = x_0^2 4\sin^2\frac{\delta}{2}\cos^2\frac{\delta}{2}; \quad a^2 2\sin^2\frac{\delta}{2} = x_0^2 4\sin^2\frac{\delta}{2}\cos^2\frac{\delta}{2},$$

and

$$a^2 = 2x_0^2\cos^2\frac{\delta}{2}.. \tag{11.19}$$

Similarly, taking into account that at the point N on the ellipse $x = -y$ (see Figure 11.7), one can obtain

$$b^2 = 2x_0^2\sin^2\frac{\delta}{2}. \tag{11.20}$$

From the expressions (11.19) and (11.20) we obtain

$$tg\frac{\delta}{2} = \frac{b}{a}. \tag{11.21}$$

Thus, in order to determine the phase shift between the voltages of the same frequency, it is sufficient to measure the half axes $a$ and $b$ of the ellipse on the oscilloscope screen. In the formula (11.13) the phase shift is denoted as $\psi$, hence

$$\psi = 2arctg\frac{b}{a}. \tag{11.21a}$$

To obtain the Lissajous figure, the voltage from the resistor $R_1$, which is proportional to the current, is applied to the input ”Y” of the oscilloscope, and the voltage from the sound generator is applied to the input "X". As a result of measurements and calculations using the formula (11.21a), we obtain the angle $\psi$ of the phase shift between the current in the circuit and the external emf.

## Procedure

The electric diagram of the experimental setup is shown in Fig. 11.8.

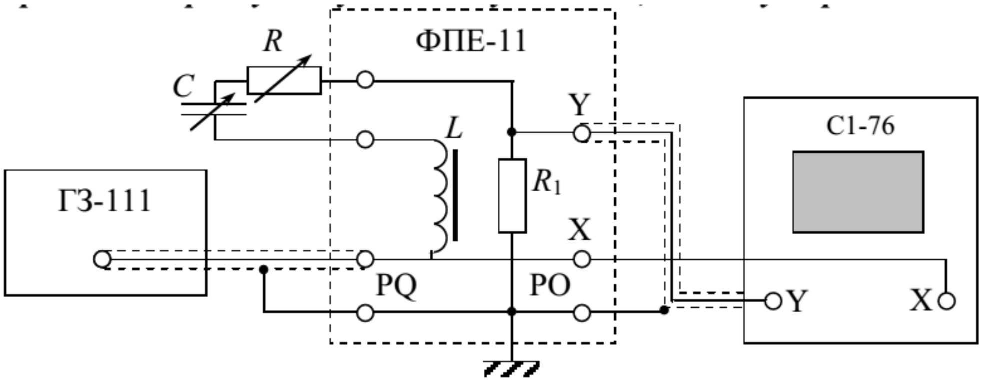



Figure 11.8.

TASK 1. Investigation of resonance curves

1. Set the capacitance $C$ = 3 $10^{-3}$ μF, and in the rheostat resistance $R$ = 1 Ω.

2. Turn on the laboratory setup and devices. Apply a signal of frequency 10 kHz from the sound generator to the input "Y" of the oscilloscope. The image of sinusoid should appear on the oscilloscope screen. Use “Brightness”, “Focus”, and “Synchronization” knobs to achieve a steady image.

3. By smoothly changing frequency of the generator, find the resonance frequency $f_{res}$ at which the amplitude of the sinusoid is maximal.

4. Rotate the "Enhancement" knobs on the front panel of the oscilloscope to make the sinusoid occupy the entire screen at the frequency $f_{res}$.

5. Set the signal frequency 2 kHz with the generator. Measure the amplitude $A$ of the sinusoid in scale divisions. Write the frequency value (2 kHz), corresponding amplitude $A$ and the amplification coefficient $K_y$, V/division to the Table 11.1.

6. Measure the sinusoid amplitudes at other frequency values in the range from 2 to 16 kHz. Vary the frequency with an interval of 1-2 kHz, but near the resonance with the interval of 0.5 kHz. Write the results into the Table 11.1.

7. Calculate the current amplitude $I_0$ in the circuit using the formula

$$I_0 = \frac{A \cdot K_y}{R_1},$$

where $R_1$ = 100 **Ω**, and add the result to the Table 11.1.

8. Set the rheostat resistance $R$ = 500 **Ω**. Make measurements according to pp. 5 - 7. Write the results in Table 11.1.

9. Set the rheostat resistance $R$ = 3000 **Ω**. Make measurements according to pp. 5 - 7. Write the results in Table 11.1.

10. Plot dependences of $I_0$ on frequency for different values of $R$ on one graph.

11. Using the graphs, determine the width of the resonance curves at the level $I_0/\sqrt{2}$ (see Fig. 11.5) and calculate the Q-factor of the circuit according to the formula $Q = f_{res}/\Delta f$.

Table 11.1

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R$ = 1 Ω | $f$, kHz | | | | | | | | | | | | | | | | | |
| | A, div | | | | | | | | | | | | | | | | | |
| | $K_y$, V/div | | | | | | | | | | | | | | | | | |
| | $I_0$ , mA | | | | | | | | | | | | | | | | | |
| $R$ = 500 Ω | $f$, kHz | | | | | | | | | | | | | | | | | |
| | A, div | | | | | | | | | | | | | | | | | |
| | $K_y$, V/div | | | | | | | | | | | | | | | | | |
| | $I_0$ , mA | | | | | | | | | | | | | | | | | |
| $R$ = 3000 Ω | $f$, kHz | | | | | | | | | | | | | | | | | |
| | A, div | | | | | | | | | | | | | | | | | |
| | $K_y$, V/div | | | | | | | | | | | | | | | | | |
| | $I_0$ , mA | | | | | | | | | | | | | | | | | |

TASK 2. Determination of dependence of the resonance frequency $f_{res}$ on the capacitance $C$.

1. Set the capacitance $C = 1\ 10^{-3}$ μF, and the rheostat resistance $R = 1\ \Omega$.

Turn the "Sweep" knob on the oscilloscope's right panel to the "X" position. An ellipse should be observed on the oscilloscope's screen.

2. By changing frequency of the sound generator achieve conversation of the ellipse into a straight line, inclined approximately at an angle of $45^0$ to the X axis. If necessary, change $K_y$. In this case, the frequency of the generator is equal to the resonance frequency $f_{\text{res}}$. Write the value of $C$ and $f_{\text{res}}$ into the Table 11.2.

3. Measure $f_{\text{res}}$ according to p. 2 for other capacitance values from $1\ 10^{-3}$ to $1\ 10^{-2}$ μF with an interval of $1\ 10^{-3}$ μF. Write results into the Table 11.2.

4. Calculate the value $z = 1/(2\pi f_{\text{рез}})^2$. Plot the dependence of the value $z$ on $C$, which should be a straight line passing through the origin. When building the graph, you should remember: accuracy of the capacitance values from the experimental setup is 5%, so you should indicate intervals on the graph, within which the value of $C$ can lie, as shown in Fig. 11.9. In such a case, the actual dependence $z$ ($C$) should lie between the lines 1 and 2, the first of which (line 1) does not go beyond the lower edges of the values of $C$, the second (line 2) does not go beyond the upper edges of the values of $C$. Build both of these lines.

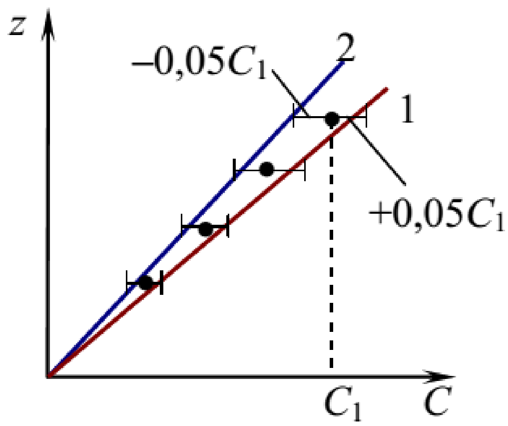



Figure 11.9.

5. Calculate the value of inductance of the solenoid as a tangent of the inclination angle of the straight line at the graph $z$ ($C$):

$$L = \frac{\Delta Z}{\Delta C}; \langle L \rangle = \frac{L_1 + L_2}{2}.$$

where $L_1$, $L_2$ are determined from lines 1 and 2, respectively.

Estimate the measurement error for $L$:

$$\delta = \frac{\Delta L}{\langle L \rangle} = \frac{L_2 - \langle L \rangle}{\langle L \rangle}$$

Table 11.2

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $C \cdot 10^9$, F | | | | | | | | | | |
| $f_{res}$, kHz | | | | | | | | | | |
| $z \cdot 10^{10}$ | | | | | | | | | | |

## Control questions

1. Derive the formula of relation between the current amplitude and the frequency of external emf in the oscillatory circuit.

2. What is the vector diagram for currents and voltages? How does the vector diagram for the series oscillatory circuit look like?

3. Derive the formula for calculating phase shift angle with help of the Lissajous figures.

4. What is resonance? How does the vector diagram look like at the resonance frequency?

5. What is the Q-factor of the oscillating circuit?

6. Prove that resonance for current takes place at the frequency of external emf $\Omega = \omega_0$.

## Recommended literature:


1. Physics for Scientists and Engineers with Modern Physics, Eighth Edition / Raymond A. Serway and John W. Jewett, Jr./ 2010 Cengage Learning Inc.
2. Кучерук І.М., Горбачук І.Т., Луцик П.П. Загальний курс фізики. Т.2. - К.:, “Техніка”, 2001.
3. Савельев И.В. Курс общей физики. Т.3. -М.: Наука,1989.
4. Сивухин Д.В. Общий курс физики. Т.3. -М.: Наука,1977.
5. Черкашин В.П. Физика. Злектричество и магнетизм. §2 - К.: Вища школа, 1986.